On condensation of charged scalars in D=3 dimensions

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Abstract

By using the gauge-invariant, but path-dependent, variables formalism, we study the impact of condensates on physical observables for a three-dimensional Higgs-like model. As a result, for the case of a physical mass term like $m_H^2\phi^*\phi$, we recover a screening potential. Interestingly enough, in the case of a "wrong-sign" mass term $-m_H^2\phi^*\phi$, unexpected features are found. It is shown that the interaction energy is the sum of an effective-Bessel and a linear potential, leading to the confinement of static charges. However, when a Chern-Simons term is included, the surprising result is that the theory describes an exactly screening phase.

1 Introduction

As well-known, a full understanding of the QCD vacuum structure and color confinement mechanism from first principles remain still elusive. However, phenomenological models still represent a key tool for understanding different non-perturbative QCD effects. Therefore, much about the physics of confinement may be learned from such models. In this connection it becomes of interest, in particular, to recall that many approaches to the problem of confinement rely on the phenomenon of condensation. For example, in the illustrative scenario of dual superconductivity [1,2,3], where it is conjectured that the QCD vacuum behaves as a dual-type II superconductor. In fact, in this case, because

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of the condensation of magnetic monopoles, the chromo-electric field acting between $q\bar{q}$ pair is squeezed into strings, and the nonvanishing string tension represents the proportionality constant in the linear potential.

On the other hand, considerable attention has been paid recently [4,5,6,7] to condensation of charged scalars and its physical consequences. The interest in studying these systems is mainly due to the possibility of describing condensed helium-4 nuclei in an electron background in white dwarf cores. More precisely, a Lorentz-violating Higgs-like effective Lagrangian has been proposed, where a nonzero vacuum expectation value for the fermion field, which permits to realize the condensation of the helium-4, plays an essential role in this development. Accordingly, the condensate characterizes the new vacuum of the theory with striking consequences over the different phases of the pure gauge sector of the proposed model. In this context, in a previous paper [8], the impact of condensates on physical observables in terms of the gauge-invariant but path-dependent variables formalism has been explored. Specifically, we have computed the static potential between test charges in a condensate of scalars and fermions. As a result, in the case of a "right-sign" mass term $m_H^2\phi^*\phi$, we have recovered the screening potential. Interestingly enough, in the case of a "wrong-sign" mass term $-m_H^2\phi^*\phi$, unexpected features were found. It was observed that the interaction energy is the sum of an effective-Yukawa and a linear potential, leading to the confinement of static charges. It is worthwhile mentioning at this point that the above static profile is analogous to that encountered in both Abelian and non-Abelian models. For example, in connection to antisymmetric tensor fields that result from the condensation of topological defects as a consequence of the Julia-Toulouse mechanism [9], in a gauge theory with a pseudoscalar coupling in the presence of a constant magnetic strength expectation value [10], and in a gauge theory which includes the mixing between the familiar photon $U(1)_{QED}$ and a second massive gauge field living in the so-called hidden-sector $U(1)_h$ [11]. Also, in the case of gluodynamics in curved space-time [12], and of a non-Abelian gauge theory with a mixture of pseudoscalar and scalar couplings, where a constant chromo-electric, or chromo-magnetic, strength expectation value is present [13]. In this way, we have provided a new correspondence among diverse effective theories. This work is devoted to study the stability of the above scenario for the three-dimensional case. Of special interest will be to check if a linearly increasing gauge potential is still present whenever we go over into three dimensions. As well as, we shall examine the effect of a Chern-Simons term, in the above scenario, on a physical observable.

It is worth recalling at this point that three-dimensional theories are interesting because of its connection to the high-temperature limit of four-dimensional theories [14,15,16,17], as well as, for their applications to condensed matter physics [18]. Thus, as already mentioned, the main purpose here is to examine the effects of the space-time dimensionality on a physical observable for the

three-dimensional case. To do this, we will work out the static potential for the model under consideration by using the gauge-invariant but path-dependent variables formalism along the lines of Ref. [8]. As we will see, there are two generic features that are common in the four-dimensional case and its lower extension studied here. First, the existence of a linear potential, leading to the confinement of static charges. However, when a Chern-Simons term is included, the surprising result is that the theory describes an exactly screening phase. The second point is related to the correspondence among diverse effective theories. In fact, in the case of a "wrong-sign" mass term $-m_H^2\phi^*\phi$, we obtain that the interaction energy is the sum of an effective-Bessel and a linear potential. Incidentally, the above static potential profile is analogous to that encountered in: a Lorentz-and CPT- violating Maxwell-Chern-Simons model [19], a Maxwell-like three-dimensional model induced by the condensation of topological defects driven by quantum fluctuations [20], a Lorentz invariant violating electromagnetism arising from a Julia-Toulouse mechanism [21], and three-dimensional gluodynamics in curved space-time [22].

Before going ahead, it is appropriate to observe here that a Abelian gauge theory possessing a confining phase may sound strange. In this context, it may be recalled that the existence of a phase structure for the continuum Abelian $U(1)$ gauge theory was obtained by including the effects due to the compactness of the $U(1)$ group, which dramatically changes the infrared properties of the model [23]. These results, first found in [23], have been ever since rederived by many different techniques [24,25,26] where the key ingredient is the contribution of self-dual topological excitations to the partition function of the theory. However, our analysis renders manifest that the mechanism of confinement in our model is not condensation of topological excitations, rather the scalars. This is what makes the current work different from earlier (above mentioned) proposals of confinement in Abelian gauge theories.

2 Three-dimensional Higgs-like model

As already stated, our principal purpose is to calculate the interaction energy between static point-like sources for a Lorentz-violating Higgs-like effective model. To this end, we shall compute the expectation value of the energy operator H in the physical state $|\Phi\rangle$, which we will denote by $\langle H\rangle_\Phi$. We begin by summarizing very quickly the recently proposed Higgs-like model [7,8], which describes a condensed of charged scalars in a neutralizing background of fermions. This would not only provide the theoretical setup for our subsequent work, but also fix the notation. The starting point is the three-dimensional

space-time Lagrangian:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + |D_\mu\phi|^2 - m_H^2\phi^*\phi + \overline{\psi}\,(i\gamma^\mu D_\mu - M)\,\psi, \tag{1}$$

where ϕ is a charged massive scalar field, A_μ is a $U\,(\,1\,)$ gauge potential, and ψ is an "heavy" fermion. The covariant derivative is defined as: $D_\mu \equiv \partial_\mu + ieA_\mu$. Let us also mention here that $m_H^2 > 0$ is a "right sign" mass term and we have not included any self-interaction for the scalar field. Following our earlier procedure [8], we shall now consider that the fermions are so heavy that they cannot be excited in the low energy regime we are studying. In such a case, the Dirac kinetic term can be neglected and the whole fermion sector of the model reduces to a constant background density J^0 coupled to A_μ, that is, $\overline{\psi}\gamma^\mu\psi \longrightarrow -\delta_0^\mu\,J^0$. This allows us to write the Lagrangian (1) as

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + |D_\mu\phi|^2 - m_H^2\phi^*\phi - eJ^0\delta_0^\mu\,A_\mu. \tag{2}$$

Once this is done, the field equations obtained by varying (2) with respect to A_μ and ϕ^* follow closely that of reference [8]:

$$\partial_\mu F^{\mu\nu} + 2e^2A^\nu|\phi|^2 = e(J_s^\nu + J^0\delta_0^\nu), \tag{3}$$

$$\Big(\,\partial_\mu\partial^\mu - e^2A_\mu A^\mu + m_H^2\,\Big)\,\phi = 0, \tag{4}$$

where $J_s^\nu \equiv i(\phi^*\partial^\nu\phi - \phi\partial^\nu\phi^*)$. In this way, the ground state of the system is described by the classical solution:

$$\overline{\psi}_0\gamma^\mu\psi_0 = -\delta_0^\mu\;J^0, \tag{5}$$

$$\phi_0 = \sqrt{\frac{J^0}{2m_H}}, \tag{6}$$

$$A_0^\mu = \frac{m_H}{e}\,\delta_0^\mu. \tag{7}$$

Once there is a non-vanishing background value for the scalar field, we choose to work in the unitary gauge, so that the phase of the ϕ - field can be gauged away. Next to this choice, we split the fields ϕ (now, $\phi = \phi^*$) and A_μ as the sum of a classical background around which there appear quantum fluctuations as it follows below:

$$\phi = \phi^* = \phi_0 + \frac{1}{\sqrt{2}}\eta\,(\,x\,)\,, \tag{8}$$

$$A_\mu = \frac{m_H}{e}\,\delta_\mu^0 + b_\mu\,(\,x\,)\,, \tag{9}$$

the corresponding Lagrangian density, up to quadratic terms in the fluctuations, is given by

$$\mathcal{L} = -\frac{1}{4}f_{\mu\nu}^2 + \frac{1}{2}\,(\partial_\mu\eta)^2 + \frac{1}{2}m_\gamma^2 b_\mu^2 + 2m_H m_\gamma b_0\,\eta\;. \tag{10}$$

where $f_{\mu\nu} \equiv \partial_\mu b_\nu - \partial_\nu b_\mu$, and $m_\gamma^2 \equiv 2e^2\phi_0^2$. Following our earlier procedure [8], integrating out the η field induces an effective theory for the b_μ field. This leads us to the following effective Lagrangian density:

$$\mathcal{L}_{eff} = -\frac{1}{4}f_{\mu\nu}^2 + \frac{1}{2}m_\gamma^2 b_\mu^2 + 2m_H^2 m_\gamma^2 \, b_0 \, \frac{1}{\Delta} \, b_0, \tag{11}$$

where $\Delta = \partial_\mu \partial^\mu$. As a consequence, the Lagrangian (1) becomes a Maxwell-Proca-like theory with a manifestly Lorentz violating term. This effective theory provide us with a suitable starting point to study the interaction energy. However, before proceeding with the determination of the energy, it is necessary to restore the gauge invariance in (11). For this purpose, we note that the Lagrangian (11) may be rewritten as

$$\mathcal{L} = -\frac{1}{4}f_{\mu\nu}^2 + \frac{1}{2}b_\mu m^2 b^\mu - \frac{1}{2}b_i \frac{(2m_H m_\gamma)^2}{\Delta} b^i, \tag{12}$$

where $m^2 \equiv m_\gamma^2\left(1 + \frac{4m_H^2}{\Delta}\right)$. With this in hand, the canonical momenta Π^μ are found to be $\Pi^0 = 0$ and $\Pi^i = -f^{0i}$. The canonical Hamiltonian is now obtained in the usual way

$$H = \int d^2x \left\{ -b_0 \left(\partial_i \Pi^i + \frac{m^2}{2} b^0 \right) - \frac{1}{2}\Pi_i \Pi^i + \frac{1}{4} f_{ij} f^{ij} - \frac{1}{2} b_i \left(m^2 - \frac{(2m_H m_\gamma)}{\Delta} \right) b^i \right\}. \tag{13}$$

Time conservation of the primary constraint ($\Pi^0 = 0$) yields a secondary constraint $\Gamma(x) \equiv \partial_i \Pi^i + m^2 b^0 = 0$. Notice that the nonvanishing bracket $\{\Pi^0, \partial_i \Pi^i + m^2 b^0\}$ shows that the above pair of constraints are second class constraints, as expected for a theory with an explicit mass term which breaks the gauge invariance. To convert the second class system into first class we enlarge the original phase space by introducing a canonical pair of fields θ and Π_θ [8]. It follows, therefore, that a new set of first class constraints can be defined in this extended space:

$$\Lambda_1 \equiv \Pi_0 + m^2 \theta, \tag{14}$$

and

$$\Lambda_2 \equiv \Gamma + \Pi_\theta. \tag{15}$$

In this way the gauge symmetry of the theory under consideration has been restored. Then, the new effective Lagrangian, after integrating out the θ field, becomes

$$\mathcal{L}_{eff} = -\frac{1}{4} f_{\mu\nu} \left[1 + \frac{m_\gamma^2}{\Delta} \left(1 + \frac{4m_H^2}{\Delta} \right) \right] f^{\mu\nu}. \tag{16}$$

Again, as was explained in [8], we observe that to get the above theory we have ignored the last term in (12) because it add nothing to the static potential calculation, as we will show it below. In other words, the new effective action

(16) provide us with a suitable starting point to study the interaction energy without loss of physical content.

We now turn our attention to the calculation of the interaction energy. In order to obtain the corresponding Hamiltonian, the canonical quantization of this theory from the Hamiltonian analysis point of view is straightforward and follows closely that of reference [8]. The canonical momenta read $\Pi^\mu = -\left[1+\frac{m_\gamma^2}{\Delta}\left(1+\frac{4m_H^2}{\Delta}\right)\right]f^{0\mu}$, and one immediately identifies the usual primary constraint $\Pi^0 = 0$ and $\Pi^i = -\left[1+\frac{m_\gamma^2}{\Delta}\left(1+\frac{4m_H^2}{\Delta}\right)\right]f^{0i}$. The canonical Hamiltonian is thus

$$H_C = \int d^2x \left\{ -b_0\partial_i\Pi^i - \frac{1}{2}\Pi_i\left[1+\frac{m_\gamma^2}{\Delta}\left(1+\frac{4m_H^2}{\Delta}\right)\right]^{-1}\Pi^i + \frac{1}{4}f_{ij}f^{ij}\right\}. \quad (17)$$

The consistency condition $\dot{\Pi}_0 = 0$ leads to the usual Gauss constraint $\Gamma_1(x) \equiv \partial_i\Pi^i = 0$. The extended Hamiltonian that generates translations in time then reads $H = H_C + \int d^2x\,(c_0(x)\Pi_0(x) + c_1(x)\Gamma_1(x))$, where $c_0(x)$ and $c_1(x)$ are the Lagrange multipliers. Since $\Pi^0 = 0$ for all time and $\dot{b}_0(x) = [b_0(x), H] = c_0(x)$, which is completely arbitrary, we discard b^0 and Π^0 because they adding nothing to the description of the system. Then, the Hamiltonian takes the form

$$H = \int d^2x \left\{ c(x)\partial_i\Pi^i - \frac{1}{2}\Pi_i\left[1+\frac{m_\gamma^2}{\Delta}\left(1+\frac{4m_H^2}{\Delta}\right)\right]^{-1}\Pi^i + \frac{1}{4}f_{ij}f^{ij}\right\}, \quad (18)$$

where $c(x) = c_1(x) - b_0(x)$. Evidently, the presence of the arbitrary quantity $c(x)$ is undesirable since we have no way to giving it a meaning in a quantum theory. As is well known, the solution to this problem is to introduce a gauge condition such that the full set of constraints become second class. A particularly convenient choice is found to be

$$\Gamma_2(x) \equiv \int\limits_{C_{\xi x}} dz^\nu b_\nu(z) \equiv \int\limits_0^1 d\lambda x^i b_i(\lambda x) = 0, \quad (19)$$

where λ $(0 \le \lambda \le 1)$ is the parameter describing the spacelike straight path $x^i = \xi^i + \lambda(x-\xi)^i$, and ξ is a fixed point (reference point). There is no essential loss of generality if we restrict our considerations to $\xi^i = 0$. The choice (19) leads to the Poincaré gauge [27]. As a consequence, we can now write down the only nonvanishing Dirac bracket for the canonical variables

$$\left\{b_i(x), \Pi^j(y)\right\}^* = \delta_i^j\delta^{(2)}(x-y) - \partial_i^x\int\limits_0^1 d\lambda x^j\delta^{(2)}(\lambda x - y). \quad (20)$$

We are now ready to find the interaction energy between point-like sources for

the model under consideration. As we have already indicated, we will calculate the expectation value of the energy operator H in the physical state $|\Phi\rangle$. In this context, we recall that the physical state $|\Phi\rangle$ can be written as

$$|\Phi\rangle \equiv \left|\overline{\Psi}\left(\mathbf{y}\right)\Psi\left(\mathbf{0}\right)\right\rangle = \overline{\psi}\left(\mathbf{y}\right)\exp\left(iq\int\limits_{\mathbf{0}}^{\mathbf{y}} dz^{i}b_{i}\left(z\right)\right)\psi\left(\mathbf{0}\right)|0\rangle\,, \tag{21}$$

where $|0\rangle$ is the physical vacuum state. The line integral is along a spacelike path starting at $\mathbf{0}$ and ending at $\mathbf{y}$, on a fixed time slice.

Next, taking into account the above Hamiltonian analysis, we then obtain

$$\langle H\rangle_{\Phi} = \langle H\rangle_{0} + \langle H\rangle_{\Phi}^{(1)}\,, \tag{22}$$

where, in this static case, $\Delta = -\nabla^2$. At the same time, $\langle H\rangle_0 = \langle 0|\,H\,|0\rangle$, and the $\langle H\rangle_{\Phi}^{(1)}$ term is given by

$$\langle H\rangle_{\Phi}^{(1)} = \langle\Phi|\int d^2x\left\{-\frac{1}{2}\Pi_i\left[1-\frac{m_\gamma^2}{\nabla^2}\left(1-\frac{4m_H^2}{\nabla^2}\right)\right]^{-1}\Pi^i+\frac{1}{4}f_{ij}f^{ij}\right\}|\Phi\rangle\,. \tag{23}$$

It should be noted that the above expression may be rewritten as

$$\langle H\rangle_{\Phi}^{(1)} = -\frac{1}{2}\frac{4M^4}{\left(M_2^2-M_1^2\right)}\int d^2x\,\langle\Phi|\,\Pi_i\left\{\alpha\frac{\nabla^2}{\left(\nabla^2-M_1^2\right)}-\beta\frac{\nabla^2}{\left(\nabla^2-M_2^2\right)}\right\}\Pi^i\,|\Phi\rangle + $$
$$+\frac{1}{4}\int d^2x\,\langle\Phi|\,f_{ij}f^{ij}\,|\Phi\rangle\,, \tag{24}$$

with $\alpha = \frac{1}{\left(M_1^2-m_\gamma^2\right)}$ and $\beta = \frac{1}{\left(M_2^2-m_\gamma^2\right)}$. While $M_1^2 = \frac{1}{2}\left(m_\gamma^2+\sqrt{m_\gamma^4-16M^4}\right)$, $M_2^2 = \frac{1}{2}\left(m_\gamma^2-\sqrt{m_\gamma^4-16M^4}\right)$ and $M\equiv\sqrt{m_\gamma m_H}$. One immediately sees that this expression is similar to that encountered in the three space dimensions case [8]. It follows, therefore, that in $(2+1)$ dimensions, the potential for two opposite charges located at $\mathbf{0}$ and $\mathbf{y}$ takes the form

$$V = -\frac{q^2}{2\pi}\frac{4M^4}{\sqrt{m_\gamma^4-16M^4}}\left[\frac{1}{M_2^2}K_0\left(M_1L\right)+\frac{1}{M_1^2}K_0(M_2L)\right], \tag{25}$$

where K_0 is a modified Bessel function, and $|\mathbf{y}|\equiv L$.

Before we proceed further, we wish to illustrate an alternative derivation of the result (25), which exhibits certain distinctive features of our methodology. To begin with, let us recall that the potential can be obtained from [28]:

$$V \equiv q\left(\mathcal{A}_0\left(\mathbf{0}\right)-\mathcal{A}_0\left(\mathbf{y}\right)\right), \tag{26}$$

where the physical scalar potential is given by

$$\mathcal{A}_0\left(x^0,\mathbf{x}\right)=\int_0^1 d\lambda x^i E_i\left(\lambda\mathbf{x}\right), \tag{27}$$

with $i=1,2$. It is worth noting here that this follows from the vector gauge-invariant field expression [29]

$$\mathcal{A}_\mu\left(x\right)\equiv A_\mu\left(x\right)+\partial_\mu\left(-\int_\xi^x dz^\mu A_\mu\left(z\right)\right), \tag{28}$$

where, as in Eq.(19), the line integral is along a spacelike path from the point ξ to x, on a fixed slice time. The gauge-invariant variables (28) commute with the sole first constraint (Gauss' law), confirming that these fields are physical variables [30]. Note that Gauss' law for the present theory reads $\partial_i\Pi^i=J^0$, where we have included the external current J^0 to represent the presence of two opposite charges. For $J^0\left(t,\mathbf{x}\right)=q\delta^{(2)}\left(\mathbf{x}\right)$ the electric field is given by

$$E^i=q\frac{4M^4}{\left(M_2^2-M_1^2\right)}\left\{\frac{1}{\left(M_1^2-m_\gamma^2\right)}\partial^i G^{(1)}\left(\mathbf{x}\right)-\frac{1}{\left(M_2^2-m_\gamma^2\right)}\partial^i G^{(2)}\left(\mathbf{x}\right)\right\}, \tag{29}$$

where $G^{(1)}\left(\mathbf{x}\right)=\frac{1}{2\pi}K_0\left(M_1\left|\mathbf{x}\right|\right)$ and $G^{(2)}\left(\mathbf{x}\right)=\frac{1}{2\pi}K_0\left(M_2\left|\mathbf{x}\right|\right)$ are the Green functions for the Proca operator in two space dimensions. Using this, the physical scalar potential, Eq.(27), reduces to

$$\mathcal{A}_0\left(t,\mathbf{x}\right)=q\frac{4M^4}{\left(M_2^2-M_1^2\right)}\left[\frac{1}{\left(M_1^2-m_\gamma^2\right)}G^{(1)}\left(\mathbf{x}\right)-\frac{1}{\left(M_2^2-m_\gamma^2\right)}G^{(2)}\left(\mathbf{x}\right)\right], \tag{30}$$

after substraction of self-energy terms. With this then, we now see that the potential for a pair of point-like opposite charges q located at $\mathbf{0}$ and $\mathbf{L}$ becomes

$$V=-\frac{q^2}{2\pi}\frac{4M^4}{\sqrt{m_\gamma^4-16M^4}}\left[\frac{1}{M_2^2}K_0\left(M_1L\right)+\frac{1}{M_1^2}K_0(M_2L)\right], \tag{31}$$

where $|\mathbf{L}|\equiv L$. It must be clear from this discussion that a correct identification of physical degrees of freedom is a key feature for understanding the physics hidden in gauge theories.

Within this framework, we now want to extend what we have done when a $m_H^2\phi^*\phi$ term and a quartic self-interaction potential is considered in expression (1), namely,

$$\mathcal{L}=-\frac{1}{4}F_{\mu\nu}^2+\left|D_\mu\phi\right|^2+m_H^2\phi^*\phi-\frac{\lambda}{6}\left(\phi^*\phi\right)^2-eJ^0\delta_\mu^0A_\mu. \tag{32}$$

As before, the last term arises from the condensation mechanism in a neutralizing background of fermions. Following the same steps that lead to (16) we

arrive at the following effective Lagrangian density:

$$\mathcal{L}_{eff} = -\frac{1}{4} f_{\mu\nu} \left[1 + \frac{m_\gamma^2}{\Delta} \left(1 + \frac{4\mu_s^2}{(\Delta + 2m_H^2)} \right) \right] f^{\mu\nu}. \tag{33}$$

In the same way as was done in the previous case, one finds

$$\begin{aligned} \langle H \rangle_\Phi = & C_1 \langle \Phi | - \frac{1}{2} \int d^2x \Pi_i \left\{ \frac{\nabla^2}{(\nabla^2 - M_2^2)} - \eta^2 \frac{\nabla^2}{(\nabla^2 - M_1^2)} \right\} \Pi^i | \Phi \rangle \\ & + C_2 \langle \Phi | \frac{1}{2} \int d^2x \Pi_i \left\{ \frac{1}{(\nabla^2 - M_2^2)} - \eta^2 \frac{1}{(\nabla^2 - M_1^2)} \right\} \Pi^i | \Phi \rangle , \end{aligned} \tag{34}$$

where $C_1 \equiv \frac{2M^4}{\left(2M^4 - m_\gamma^2\right)}$, $C_2 \equiv \frac{4m_H^2 M^4}{\left(2M^4 - m_\gamma^2\right)}$, and $\eta^2 \equiv \frac{m_\gamma^2}{2m_H^2}$. While $M_1^2 = m_\gamma^2$, $M_2^2 = 2m_H^2$, and $M = \sqrt{m_\gamma m_H}$.

According to our earlier procedure, we find that the potential for two opposite charges located at $\mathbf{0}$ and $\mathbf{y}$ takes the form

$$V = -\frac{q^2}{2\pi} C_1 \left\{ K_0 (M_2 L) - \frac{M_1^2}{M_2^2} K_0 (M_1 L) \right\} + \frac{q^2}{4} \frac{C_2}{M_2} \left\{ 1 - \frac{M_1}{M_2} \right\} L. \tag{35}$$

Here, in contrast to the previous case, unexpected features are found. In fact, we see that the static potential profile displays the conventional screening part, encoded in the modified Bessel function, and the linear confining potential.

3 Three-dimensional Higgs-like model and a Chern-Simons term

We now pass on to the calculation of the interaction energy between static pointlike sources for the $(2+1)$-dimensional Higss-like model with a Chern-Simons term. In other words, in this section we concentrate on the effect of including the Chern-Simons term in the confinement and screening nature of the potential. With this in mind, we start by writing:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 + \frac{s}{2} \varepsilon^{\nu\kappa\lambda} A_\nu \partial_\kappa A_\lambda + |D_\mu \phi|^2 - m_H^2 \phi \phi^* - e J^0 \delta_0^\mu A_\mu. \tag{36}$$

Proceeding as in the previous subsection, the effective Lagrangian is given by

$$\mathcal{L}_{eff} = -\frac{1}{4} f_{\mu\nu} \left[1 + \frac{m_\gamma^2}{\Delta} \left(1 + \frac{4m_H^2}{\Delta} \right) \right] f^{\mu\nu} + \frac{s}{2} \varepsilon^{\mu\nu\lambda} b_\mu \partial_\nu b_\lambda. \tag{37}$$

The effective Lagrangian expressed by (37) describes the effective dynamics of the quantum b_μ-field. Since we are interested in pursuing an investigation

of the potential which comes from the b_μ-field exchange, we can say that we are actually restricting our analysis to the low-frequency regime of $\mathcal{L}_{eff}$. In this region, it is legitimate to drop the $f_{\mu\nu}f^{\mu\nu}$-term respect to the other terms, the reason being that this term is quadratic in the frequencies and, therefore, the terms m_γ^2 and s dominate. The space-time dependence of b_μ and, hence, its dynamics, is accounted for in the $f_{\mu\nu}^2$ and in the Chern-Simons terms. Considering the regime of low-frequencies, it is true that they are both much smaller than the term in m^2. However, disregarding them simultaneously would lead us to a completely different regime, where only constant field configurations would be considered. To ensure that contributions from non-constant configurations are also taken into account, we have to keep at the least the Chern-Simons term, since it is linear in the frequency whereas the Maxwell-term is quadratic. So, our claim is that the s-term is the one that survives in the low-frequency regime, and this guarantees that non-constant field configurations are not thrown away. Therefore, keeping in mind that we are bound to the low-frequency regime, we can express $\mathcal{L}_{eff}$ as follows:

$$\mathcal{L}_{eff} = -\frac{1}{4}f_{\mu\nu}\left[\frac{m_\gamma^2}{\Delta}\left(1+\frac{4m_H^2}{\Delta}\right)\right]f^{\mu\nu} + \frac{s}{2}\varepsilon^{\mu\nu\lambda}b_\mu\partial_\nu b_\lambda. \tag{38}$$

It is now once again straightforward to apply the gauge-invariant formalism discussed in the preceding section. For this purpose, we start by observing that the canonical momenta read $\Pi^\mu = \left[\frac{m_\gamma^2}{\Delta}\left(1+\frac{4m_H^2}{\Delta}\right)\right]f^{\mu 0} + \frac{s}{2}\varepsilon^{0\mu\nu}b_\nu$. As we can see there is one primary constraint $\Pi^0 = 0$, and $\Pi^i = \left[\frac{m_\gamma^2}{\Delta}\left(1+\frac{4m_H^2}{\Delta}\right)\right]f^{i0} + \frac{s}{2}\varepsilon^{ij}b_j$. The canonical Hamiltonian for this system, in terms of $B = \varepsilon_{ij}\partial^i b^j$ and $E^i = \left[\frac{m_\gamma^2}{\Delta}\left(1+4\frac{m_H^2}{\Delta}\right)\right]^{-1}(\Pi^i - \frac{s}{2}\varepsilon^{ij}b_j)$, is in this case

$$\begin{aligned} H_C = &\int d^2x\left\{-b^0\left(\partial_i\Pi^i + \frac{s}{2}B\right) + \frac{1}{2}E^i\left[\frac{m_\gamma^2}{\Delta}\left(1+\frac{4m_H^2}{\Delta}\right)\right]E^i\right\} \\ &+\int d^2x\left\{\frac{1}{2}B\left[\frac{m_\gamma^2}{\Delta}\left(1+\frac{4m_H^2}{\Delta}\right)\right]B\right\}. \end{aligned} \tag{39}$$

The conservation in time of the primary constraint Π^0 leads to the secondary constraint $\Gamma_1(x) \equiv \partial_i\Pi^i + \frac{s}{2}B = 0$. The above constraints are the first-class constraints of the theory since no more constraints are generated by the time preservation of the secondary constraints. Once again, the corresponding total (first-class) Hamiltonian that generates the time evolution of the dynamical variables reads $H = H_C + \int d^2x\,(c_0(x)\,\Pi_0(x) + c_1(x)\,\Gamma_1(x))$, where $c_0(x)$ and $c_1(x)$ are the Lagrange multiplier fields to implement the constraints. As before, neither $b_0(x)$ nor $\Pi_0(x)$ are of interest in describing the system and may be discarded from the theory. As a result the Hamiltonian becomes

$$
\begin{aligned}
H = & \int d^2x \left\{ c(x) \left(\partial_i \Pi^i + \frac{s}{2} B \right) + \frac{1}{2} E^i \left[\frac{m_\gamma^2}{\Delta} \left(1 + \frac{4m_H^2}{\Delta} \right) \right] E^i \right\} \\
& + \int d^2x \left\{ \frac{1}{2} B \left[\frac{m_\gamma^2}{\Delta} \left(1 + \frac{4m_H^2}{\Delta} \right) \right] B \right\},
\end{aligned} \tag{40}
$$

where $c(x) = c_1(x) - b_0(x)$. Since our main motivation is to compute the static potential, we will adopt the same gauge-fixing condition that was used in the last subsection. Thus, in order to illustrate the discussion, we now write the Dirac brackets in terms of the magnetic and electric fields as:

$$
\{E_i(x), E_j(y)\}^* = -s \left[\frac{m_\gamma^2}{\Delta} \left(1 + \frac{4m_H^2}{\Delta} \right) \right]^{-2} \varepsilon_{ij} \delta^{(2)}(x-y), \tag{41}
$$

$$
\{B(x), B(y)\}^* = 0, \tag{42}
$$

$$
\{E_i(x), B(y)\}^* = - \left[\frac{m_\gamma^2}{\Delta} \left(1 + \frac{4m_H^2}{\Delta} \right) \right]^{-1} \varepsilon_{ij} \partial_x^j \delta^{(2)}(x-y). \tag{43}
$$

One can now easily derive the equations of motion for the electric and magnetic fields. We find

$$
\dot{E}_i(x) = -s \left[\frac{m_\gamma^2}{\Delta} \left(1 + \frac{4m_H^2}{\Delta} \right) \right]^{-1} \varepsilon_{ij} E_j(x) - \varepsilon_{ij} \partial^j B(x), \tag{44}
$$

$$
\dot{B}(x) = -\varepsilon_{ij} \partial^j E_i(x). \tag{45}
$$

In the same way, we write the Gauss law as

$$
\left[\frac{m_\gamma^2}{\Delta} \left(1 + \frac{4m_H^2}{\Delta} \right) \right] \partial_i E^i + sB + J^0 = 0. \tag{46}
$$

As before, we have included the external current J^0 to represent the presence of two opposite charges. For $J^0(t, \mathbf{x}) = q\delta^{(2)}(\mathbf{x})$ the electric field, in the $m_H^2 \gg \mathbf{k}^2$ case, is given by

$$
E^i = \frac{q}{m_\gamma^2 \sqrt{1 - 32s^2 m_H^2 / m_\gamma^4}} \left[\alpha \partial^i G^{(2)}(\mathbf{x}) - \beta \partial^i G^{(1)}(\mathbf{x}) \right], \tag{47}
$$

where $\alpha = M_2^2 - 4m_H^2$, $\beta = M_1^2 - 4m_H^2$, $M_1^2 = \frac{m_\gamma^4}{2s^2} \left[1 + \sqrt{1 - 32s^2 m_H^2 / m_\gamma^4} \right]$, and $M_2^2 = \frac{m_\gamma^4}{2s^2} \left[1 - \sqrt{1 - 32s^2 m_H^2 / m_\gamma^4} \right]$. Again, $G^{(1)}(\mathbf{x}) = \frac{1}{2\pi} K_0(M_1 |\mathbf{x}|)$, and

$G^{(2)}(\mathbf{x}) = \frac{1}{2\pi}K_0(M_2|\mathbf{x}|)$. Combining Eqs. (47) and (26), we can write immediately the potential for a pair of point-like opposite charges q located at $\mathbf{0}$ and $\mathbf{L}$, as

$$V = -\frac{q^2}{2\pi}\frac{1}{m_\gamma^2\sqrt{1-32s^2m_H^2\big/m_\gamma^4}}\left[\alpha K_0\left(M_2L\right)-\beta K_0(M_1L)\right], \tag{48}$$

where $|\mathbf{L}| = L$.

Let us consider next the effect of a $m_H^2\phi^*\phi$ term and a quartic self-interaction potential in expression (36), that is,

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + \frac{s}{2}\varepsilon^{\nu\kappa\lambda}A_\nu\partial_\kappa A_\lambda + |D_\mu\phi|^2 + m_H^2\phi^*\phi - \frac{\lambda}{6}(\phi^*\phi)^2 - eJ^0\delta_0^\mu A_\mu. \tag{49}$$

Again, in the same way as was done in the previous case, one finds

$$\mathcal{L}_{eff} = -\frac{1}{4}f_{\mu\nu}\left[1+\frac{m_\gamma^2}{\Delta}\left(1+\frac{4\mu_s^2}{(\Delta+2m_H^2)}\right)\right]f^{\mu\nu} + \frac{s}{2}\varepsilon^{\mu\nu\lambda}b_\mu\partial_\nu b_\lambda. \tag{50}$$

Again, as discussed in going from Eq. (37) to Eq. (38), we here also work in the regime of low frequencies, so that it the $f_{\mu\nu}f^{\mu\nu}$-term can be neglected in comparison with the other terms.

$$\mathcal{L}_{eff} = -\frac{1}{4}f_{\mu\nu}\left[\frac{m_\gamma^2}{\Delta}\left(1+\frac{4\mu_s^2}{(\Delta+2m_H^2)}\right)\right]f^{\mu\nu} + \frac{s}{2}\varepsilon^{\mu\nu\lambda}b_\mu\partial_\nu b_\lambda. \tag{51}$$

Once this is done, the above Hamiltonian constrained analysis can be repeated step by step for this effective theory. Accordingly, the potential for a pair of point-like opposite charges q located at $\mathbf{0}$ and $\mathbf{L}$, in the ${}^{\mu_s^2}\!/_{m_H^2} \to 0$ case, is given by

$$V = \frac{q^2}{2\pi}K_0(ML), \tag{52}$$

where $M^2 = {}^{m_\gamma^4}\!\big/_{s^2}$. We immediately see that, unexpectedly, the confining potential between static charges vanishes in this case.

4 Final remarks

To conclude, this work is a sequel to [8], where we have considered a three-dimensional extension of the recently proposed Higgs-like model [7], which describes a condensed of charged scalars in a neutralizing background of

fermions. To do this, we have exploited a crucial point for understanding the physical content of gauge theories, namely, the correct identification of field degrees of freedom with observable quantities. It was shown, that for the case of a term physical mass $m_H^2 \phi^* \phi$, a screening potential is recovered. Interestingly enough, in the case of a "wrong-sign" mass term $-m_H^2 \phi^* \phi$, unexpected features were found. It was observed that the interaction energy is the sum of an effective-Bessel and a linear potential, leading to the confinement of static charges. However, when a Chern-Simons term is included, the surprising result is that the theory describes an exactly screening phase.

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